An Extramural Seminar on "Linear Transformation: An Essential Tool of Linear Algebra"

By Dr Arun Kumar Tripathy,
Lecturer in Mathematics,
SSB College, Mahakalpara
on 18.11.2017

Organised by Department of Mathematics

Pattamundai College, Pattamundai

REPORT

A seminar was organised by Department of Mathematics, Pattamundai College, Pattamundai on 18.11.2017 on the topic "**LINEAR TRANSFORMATION :AN ESSENTIAL TOOL OF LINEAR ALGEBRA**". Dr Arun Kumar Tripathy , Lecturer in Mathematics, S.S.B College, Mahakalapada who graced the seminar with his analytical thinking. We were able to get the beautiful glimpses of the students of our Department. Sri Arabinda Pandab, Head of the Department gave a key note address of the topic and welcomed the guests on the diace and the participants. The meeting was ended with a vote of thanks by Dr Nirmal Kumar Sahoo, another faculty member.

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Linear Transformations: An Essential Tool of Linear Algebra

- Dr Arun Kumar Tripathy, Lecturer in Mathematics,
SSB College, Mahakalpara

Theorem. *Let V and W be vector spaces over the field F. Let T and U be two linear transformations from V into W. The function* $(T+U)$ *defined point wise by*

$$(T+U)(\underline{v}) = T\underline{v} + U\underline{v}$$

is a linear transformation from V into W. Furthermore, if $s \in F$, *the function* (sT) *defined by*

$$(sT)(\underline{v}) = s(T\underline{v})$$

is also a linear transformation from V into W. The set of all linear transformation from V into W, together with the addition and scalar multiplication defined above, is a vector space over the field F.

Proof. Suppose that T and U are linear transformation from V into W. For $(T+U)$ defined above, we have

$$\begin{aligned}(T+U)(s\underline{v}+\underline{w}) &= T(s\underline{v}+\underline{w}) + U(s\underline{v}+\underline{w}) \\ &= s(T\underline{v}) + T\underline{w} + s(U\underline{v}) + U\underline{w} \\ &= s(T\underline{v}+U\underline{v}) + (T\underline{w}+U\underline{w}) \\ &= s(T+U)\underline{v} + (T+U)\underline{w},\end{aligned}$$

which shows that $(T+U)$ is a linear transformation.
Similarly, we have

$$\begin{aligned}(rT)(s\underline{v}+\underline{w}) &= r(T(s\underline{v}+\underline{w})) \\ &= r(s(T\underline{v}) + (T\underline{w})) \\ &= rs(T\underline{v}) + r(T\underline{w}) \\ &= s(r(T\underline{v})) + rT(\underline{w}) \\ &= s((rT)\underline{v}) + (rT)\underline{w}\end{aligned}$$

which shows that (rT) is a linear transformation.

To verify that the set of linear transformations from V into W together with the operations defined above is a vector space, one must directly check the conditions

We denote the space of linear transformations from V into W by $L(V, W)$. Note that $L(V, W)$ is defined only when V and W are vector spaces over the same field. □

Fact. *Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then the space $L(V, W)$ is finite-dimensional and has dimension mn.*

Theorem. *Let V, W, and Z be vector spaces over a field F. Let $T \in L(V, W)$ and $U \in L(W, Z)$. Then the composed function UT defined by $(UT)(\underline{v}) = U(T(\underline{v}))$ is a linear transformation from V into Z.*

Proof. Let $\underline{v}_1, \underline{v}_2 \in V$ and $s \in F$. Then, we have

$$\begin{aligned}(UT)(s\underline{v}_1 + \underline{v}_2) &= U(T(s\underline{v}_1 + \underline{v}_2)) \\ &= U(sT\underline{v}_1 + T\underline{v}_2) \\ &= sU(T\underline{v}_1) + U(T\underline{v}_2) \\ &= s(UT)(\underline{v}_1) + (UT)(\underline{v}_2),\end{aligned}$$

as desired. □

Definition. *If V is a vector space over the field F, a **linear operator** on V is a linear transformation from V into V*

Definition. *A linear transformation T from V into W is called **invertible** if there exists a function U from W to V such that UT is the identity function on V and TU is the identity function on W. If T is invertible, the function U is unique and is denoted by T^{-1}. Furthermore, T is invertible if and only if*

1. *T is one-to-one: $T\underline{v}_1 = T\underline{v}_2 \Longrightarrow \underline{v}_1 = \underline{v}_2$*
2. *T is onto: the range of T is W.*

Example 5.1.6. *Consider the vector space V of semi-infinite real sequences $\mathbb{R}^\omega$ where $\underline{v} = (v_1, v_2, v_3, \ldots)$ V with $v_n \in \mathbb{R}$ for $n \in \mathbb{N}$. Let $L : V \to V$ be the left-shift linear transformation defined by*

$$L\underline{v} = (v_2, v_3, v_4, \ldots\ldots)$$

and $R : V \to V$ be the right-shift linear transformation defined by

$$R\underline{v} = (0, v_1, v_2, \ldots\ldots).$$

Notice that L is onto but not one-to-one and R is one-to-one but not onto. Therefore, neither transformation is invertible.

Example . *Consider the normed vector space V of semi-infinite real sequences R^{ω} with the standard Schauder basis $\{\underline{e}_1, \underline{e}_2, \ldots\}$. Let $T : V \to V$ be the linear transformation that satisfies $T\underline{e}_i = i^{-1}\underline{e}_i$ for $i = 1, 2,$ Let the linear transformation $U : V \to V$ satisfy $U\underline{e}_i = i\underline{e}_i$ for $i = 1, 2, \ldots$. It is easy to verify that $U = T^{-1}$ and $UT = TU = I$.*

This example should actually bother you somewhat. Since T reduces vector components arbitrarily, its inverse must enlarge them arbitrarily. Clearly, this is not a desirable property. Later, we will introduce a norm for linear transforms which quantifies this problem.

Theorem . *Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If T is invertible, then the inverse function T^{-1} is a linear transformation from W onto V.*

Proof. Let $\underline{w}_1$ and $\underline{w}_2$ be vectors in W and let $s \in F$. Define $\underline{v}_j = T^{-1}\underline{w}_j$, for $j = 1, 2$. Since T is a linear transformation, we have

$$T(s\underline{v}_1 + \underline{v}_2) = sT(\underline{v}_1) + T(\underline{v}_2) = s\underline{w}_1 + \underline{w}_2.$$

That is, $s\underline{v}_1 + \underline{v}_2$ is the unique vector in V that maps to $s\underline{w}_1 + \underline{w}_2$ under T. It follows that

$$T^{-1}(s\underline{w}_1 + \underline{w}_2) = s\underline{v}_1 + \underline{v}_2 = s\,T^{-1}\underline{w}_1 + T^{-1}\underline{w}_2$$

and T^{-1} is a linear transformation. □

Definition:

A **homomorphism** is a mapping between algebraic structures which preserves all relevant structure. An **isomorphism** is a homomorphism which is also invertible. For vector spaces, the relevant structure is given by vector addition and scalar multiplication. Since a linear transformation preserves both of these operation, it is also a *vector space homomorphism*. Likewise, an invertible linear transformation is a *vector space isomorphism*.

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To

Dr Arun Kumar Tripathy,
Lecturer in Mathematics,
S.S.B College, Mahakalpara.

Sub: - An invitation as Resource Person in the Extramural Seminar in Department of Mathematics.

Sir,

It is my pleasure to invite you as **Resource Person** in the Extramural Seminar on the topic **"Linear Transformation: An Essential Tool of Linear Algebra"** to be organiged by Department of Mathematics, at 11.00 am on 18th November 2017 in our institution.

Your kind presence for this occasion is highly solicited.

Yours Faithfully,

18/11/17

Principal
Pattamundai College,
Pattamundai

Principal
Pattamundai College

An Extramural Seminar on "Linear Transformations: An Essential Tool of Linear Algebra"

Department of Mathematics

Pattamundai College, Pattamundai

Dt. 18.11.2017

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Organised By: DEPARTMENT OF MATHEMATICS
PATTAMUNDAI DEGREE COLLEGE, PATTAMUNDAI


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